Fourth International Conference on FRP Composites in Civil Engineering (CICE2008)
22-24July 2008, Zurich, Switzerland

Evaluation of crack spacing in RC elements externally bonded with FRP

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ABSTRACT: A critical review of the available code provisions to evaluate crack width and spacing in RC elements strengthened with FRP sheets is dealt with in this paper. Many experimental results of tests performed by the Authors on RC beams and ties externally bonded with FRP are summarized and compared with the code provisions in terms of crack width and spacing. A new formulation to evaluate crack spacing in externally reinforced RC elements has been calibrated on the experimental results and proposed to be introduced in the expression of crack width provided by Eurocode 2.

1 INTRODUCTION

Strengthening of existing RC structures by external bonded reinforcement (EBR) made of Fiber Reinforced Plastic (FRP) materials requests to satisfy both ultimate and serviceability conditions. In particular at serviceability conditions the FRP EBR modifies the cracking development due to the high contribution of the laminate in transferring tensile stresses to the concrete; number of cracks and crack spacing change depending on the bond behavior at the concrete-laminate interface, i.e. the crack spacing reduces and the crack width can not increase.

Few experimental results are available in the technical literature about cracking phenomena in FRP EBR RC elements and the existing code indications are based on the formulations usually adopted for RC elements. Therefore the evaluation of crack spacing in FRP EBR RC elements is a relevant topic and can also influence ultimate conditions when debonding occurs at intermediate cracks (Chen et al. 2006). Basing on the new formulations furnished in last Eurocode2 (2004) to evaluate crack widths and spacing in RC elements, a new proposal to calculate crack spacing in the FRP EBR RC ones is presented, basing on statistical elaborations of the results belonging to experimental tests realized by the Authors and by other researchers on FRP EBR RC beams and ties. The proposed expression for crack spacing is introduced in the procedure suggested by Eurocode 2 (2004) to evaluate crack width in RC elements.

2 CODE FORMULATIONS FOR CRACKING VERIFICATIONS

The past version of Eurocode2 (1992), neglecting the contribution of concrete strain in tension, suggested formulations to calculate the average strain in steel, ε_{sm}, and the mean crack spacing, s_{rm}, of a generic RC element in tension, in order to evaluate the mean and characteristic values of crack widths, w_m and w_k, according to the following approach:

$$w_m = \varepsilon_{sm} \cdot s_{rm} \quad (1a) \qquad w_k = \beta \cdot \varepsilon_{sm} \cdot s_{rm} \quad (1b)$$

$$\varepsilon_{sm} = \varepsilon_s \left[1 - \beta_1 \beta_2 \left(\frac{\sigma_{cr}}{\sigma_s} \right)^2 \right] \qquad (2) \qquad\qquad s_{rm} = 50 + 0.25 \cdot k_1 \cdot k_2 \cdot \frac{\phi}{\mu_s} \quad [\text{mm}] \qquad (3)$$

where β is a factor equal to 1.7 relating the mean value of crack width to the design one, β_1 takes in account the bond characteristics of the internal reinforcement (1 for ribbed bars and 0.5 for smooth bars) and β_2 considers the loading type (1 for short-term loading and 0.5 for long-term loading); σ_{cr} is the tensile stress in the steel bar at the first cracking load, σ_s and ε_s are stress and strain in the steel bar at the cracked section at the service load. The approach is extended to RC beams by a suitable evaluation of steel stress.

Crack spacing depends on the diameter of steel bar, ϕ, and the ratio, μ_s, of the internal steel reinforcement to the effective area of concrete in tension, $A_{c,eff}$. For flexural elements Eurocode2 (1992) suggests to calculate $A_{c,eff}$ as the minimum value between $(2.5 \cdot B \cdot c)$ and $(B \cdot (H - x_c)/3)$, being B, H, c, x_c width, height, concrete cover and neutral axis (at cracked condition) of the section. Finally k_1 is a bond coefficient (0.8 for ribbed and 1.6 for smooth steel bars), while k_2 takes into account type loading (0.5 for flexural and 1 for tensile load).

Now in Eurocode 2 (2004) the design value of crack width is directly evaluated as:

$$w_k = s_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm}) \qquad (4)$$

being $s_{r,max}$ the maximum crack spacing and $(\varepsilon_{sm} - \varepsilon_{cm})$ the difference of the mean strain of steel to the concrete between cracks, to be calculated as follows:

$$s_{r,max} = 3.4 \cdot c + 0.425 \cdot k_1 \cdot k_2 \cdot \frac{\phi}{\mu_s} \quad [\text{mm}] \qquad (5)$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s}{E_s} \cdot - k_t \left[\frac{f_{ctm} \cdot A_{c,eff}}{E_s \cdot A_s} + \frac{f_{ctm}}{E_{cm}} \right] \qquad (6)$$

being E_s and E_{cm} the Young's modulus of steel and concrete, f_{ctm} the mean tensile strength of concrete, A_s the internal steel reinforcement in tension, k_t a factor related to load duration (0.6 for short and 0.4 for long term loading). In this new proposal the effect of the surface type of bars is neglected in evaluating the mean strain. Coefficients have been defined based on experimental tests in serviceability conditions, i.e. in a small range of variability of steel stresses allowing assuming a stabilized state of the cracking pattern. In fact both expressions of EC2 (1992, 2004) for crack spacing are independent from steel stress level, but in presence of an externally bonded FRP strengthening, the serviceability conditions can be enhanced to a larger field of variability of steel stress, even to yielding condition. Therefore considering that the steel stress varies according to the amount of FRP reinforcement, the cracking pattern is not a stable condition and the factors obtained by experimental tests could be not significant anymore.

A specific formulation for crack spacing in FRP EBR RC elements, not depending on the steel stress level because has to be used in Eqn.(1) with Eqn.(2), is suggested in the fib bulletin 14 (2001):

$$s_{rm} = \frac{2 \cdot f_{ctm} \cdot A_{c,eff}}{\tau_{fm} \cdot u_f + \tau_{sm} \cdot u_s} \qquad \tau_{s,m} = 1.8 \cdot f_{ctm} \qquad \tau_{f,m} = 1.25 \cdot f_{ctm} \qquad (7)$$

being u_s and u_f the bond perimeter of steel bar and FRP laminates [mm], t_f the thickness of FRP [mm], $\tau_{s,m}$ and $\tau_{f,m}$ the bond stresses [MPa] along the steel-concrete and the FRP-concrete interfaces assumed constant in s_{rm}.

3 COMPARISON BETWEEN EXPERIMENTAL DATABASE AND CODE PROVISIONS

The experimental program on RC beams externally strengthened with carbon FRP sheets, used herein as database for analysis of cracking phenomena, is comprehensive of four points bending tests on 32 beams, belonging to various experimental programs (three batches) which results have been already described in detail in other papers (Ceroni et al. 2004; Ceroni and Pecce 2005; Ceroni et al. 2006). Batches are different for dimensions of section (100mm x 180mm for batch 1, 100mm x 150mm or 150mm x 100mm for batch 2, 200mm x 400mm for batch 3), internal steel reinforcement (0.87% and 1.26% for batch 1, 0.67% and 1.05% for batch 2, 0.15% for batch 3) and span length

(1.8 for batches 1 and 2, 3.4 for batch 3). Specimens without external strengthening (equivalent reinforcement percentage variable from 0.67% to 2.35%) were tested as control elements for each series in order to distinguish the effect of the FRP externally bonded flexural strengthening.

The experimental program on tie-specimens (square section 100mm side, 1200mm long), discussed in detail in (Ceroni et al. 2004), includes three series, made of a reference not strengthened tie and three specimens externally reinforced on two opposite sides with carbon or glass FRP layers with a central steel bar (diameter 10mm or 14mm) as internal reinforcement.

To analyze cracking at serviceability conditions, both $s_{rm,exp}$ and w_{exp} are referred to two load levels corresponding to tensile stress in the steel reinforcement equal to about 280 and 400MPa. For the beams measures are referred to the part stressed by constant bending moment and crack widths have been evaluated as average values measured by the mechanical devices or the LVDTs.

In general at each load level, $s_{rm,exp}$ is defined as the ratio of the distance between the cracks at the ends of the constant bending moment length to the number of cracks, n_c, reduced of one.

In Figures 1-3 the experimental values of crack spacing, $s_{rm,exp}$, and width, w_{exp}, of all tests, are compared with the design values provided by EC2 (1992), EC2 (2004) and fib bulletin 14 (2001). In both EC2 formulations of crack spacing (Eqns. 3 and 5) the FRP external reinforcement is taken into account varying the expression of the ratio μ_s:

$$\mu_s = \frac{A_s + A_f \cdot E_f / E_s}{A_{c,eff}} \tag{8}$$

The effective area of concrete in tension is evaluated according to EC2 for beams, while for ties the area having a radius of 3 times the diameter of the steel bar has been assumed.

Crack spacing are generally overestimated by EC2 formulations and underestimated by fib bulletin, as shown in the graphs of Figures 1a, 2a, 3a. The comparisons between experimental and code values evidence that the new EC2 (2004) formulation (Eqn.5), introducing the maximum crack spacing, overestimates the experimental values respect to the previous one (EC2, 1992) (Eqn.3) referring to mean values. Furthermore about evaluation of design value of crack width, w_k, in Figures 1b, 2b, 3b the comparison between code provisions and experimental results are reported; in particular for the past EC2 (1992) and fib bulletin (2001) the theoretical crack spacing (Eqns.3-7) have been used in Eqn.1b adopting the coefficient β=1.7, while for the new EC2 (2004) values given by Eqn.5 are introduced in Eqn.4 that directly furnishes design provisions.

Both EC2 formulations are safe in most cases, while the fib formulation often underestimates the experimental crack widths due to the lower values of predicted crack spacing.

For each formulation, the mean percentage deviation, $\sigma_{\%}$, and the absolute one, σ, of code provisions respect to experimental results have been calculated as follows and reported in Table 1:

$$\sigma_{\%} = \sqrt{\frac{\sum_{i=1}^{n}\left(\frac{w_{code,i} - w_{exp,i}}{w_{exp,i}}\right)^2}{n-1}} \qquad \sigma = \sqrt{\frac{\sum_{i=1}^{n}\left(w_{code,i} - w_{exp,i}\right)^2}{n-1}} \tag{9}$$

being $w_{code,i}$ and $w_{exp,i}$ respectively the code provision and the corresponding experimental result, n the total number of measured data available. Therefore a variable δ, its mean value and standard deviation are defined as follows and reported in Table 1:

$$\delta_i = \frac{w_{code,i}}{w_{exp,i}} \qquad \overline{\delta} = \frac{1}{n}\sum_{i=1}^{n}\delta_i \qquad \sigma_\delta = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\delta_i - \overline{\delta})^2} \tag{10}$$

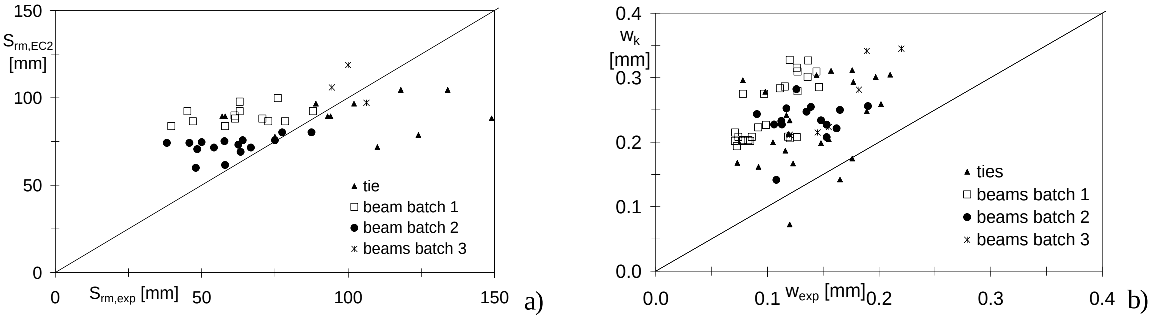

Fig. 1. Experimental and code values EC2 (1992): a) crack spacing; b) crack width

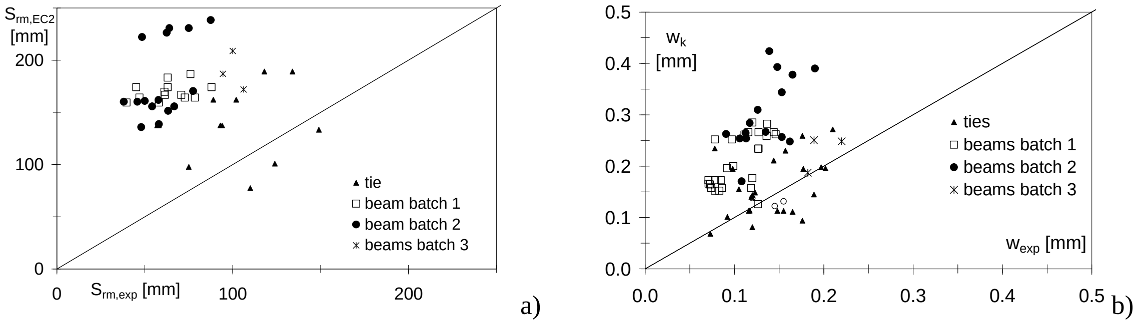

Fig. 2. Experimental and code values EC2 (2004): a) crack spacing; b) crack width

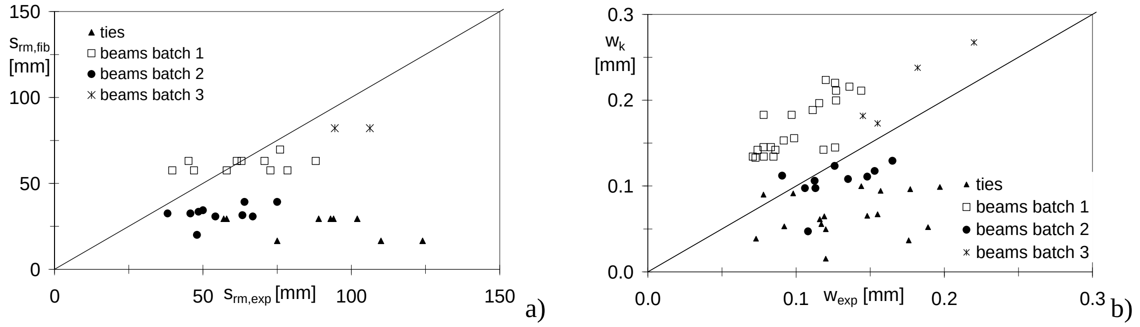

Fig. 3. Experimental and code values fib bulletin 14 (2001): a) crack spacing; b) crack width

4 NEW FORMULATION PROPOSED

The experimental results have been used to calibrate the following new formulation for crack spacing in RC elements externally strengthened with FRP materials, in order to adopt the approach of EC2 (1992, 2004) to calculate crack width according to Eqns.1-4:

$$s_{rm,th} = s_0 + k \cdot \frac{A_{c,eff}{}^{\gamma} \cdot \phi^{\alpha}}{A_s{}^{\delta} + \left(\frac{A_f E_f}{E_s}\right)^{\beta}} \qquad [\text{mm}] \qquad (11)$$

Considering the results of the experimental database previously described, a calibration of the coefficients reported in Eqn.11 has been developed providing the following values best fitting the experimental results: s_0 = 20mm, k = 4; α = 1; β = 0.75; γ = 0.5; δ = 0.75.

In Fig.4a the comparison between the experimental data and the theoretical provisions of Eqn.11 is reported. The values of percentage and absolute deviation listed in Table 2 evidence a reduction respect to the others formulations previously examined in terms of mean values (35% and 22mm vs. 42% and 25mm, 47% and 41mm respectively for Eqn.3 and Eqn.7).

The variable δ and the corresponding mean and standard deviation values, defined as follows, result:

$$\delta_i = \frac{s_{rm,exp,i}}{s_{rm,th,i}} \qquad \overline{\delta} = \frac{1}{n}\sum_{i=1}^{n}\delta_i = 0.94 \qquad \sigma_{\delta} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\delta_i - \overline{\delta})^2} = 0.273 \qquad (12)$$

Because it has been verified that variable δ has a lognormal distribution, its theoretical 95th percentile has been calculated in order to obtain the corresponding value of variable δ: δ_k=1.45.

Assuming that the values $s_{rm,th}$, given by Eqn.11, represent mean provisions of crack spacing, the characteristic values can be defined as follows:

$$s_{rm,th,k} = s_{rm,th} \cdot \frac{\delta_k}{\overline{\delta}} = s_{rm,th} \cdot \frac{1.45}{0.94} \approx 1.55\, s_{rm,th} \qquad (13)$$

In Fig.4b the characteristic values given by Eqn.13 are compared with the experimental results, and in Table 2 the mean deviations of proposed expression (Eqn.13) respect to the experimental results are reported, evidencing lower absolute and percentage scatters respect to the characteristic code provisions (Eqn.4) given by new EC2 (2004) (92% and 53.6mm vs. 175% and 103mm).

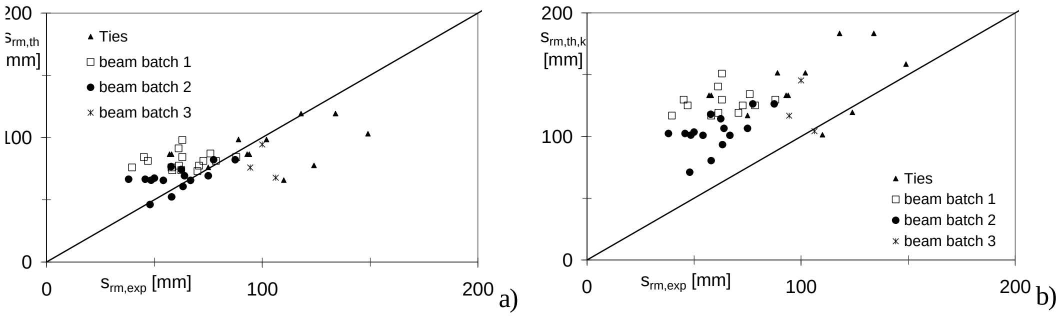

Fig. 4. Experimental vs. theoretical crack spacing: a) mean value (Eqn.11); characteristic values (Eqn.13)

The mean value of crack spacing, $s_{rm,th,}$, evaluated by the proposed formulation (Eqn.11) can be introduced in Eqn.1b to calculate the mean crack width as follows:

$$w_k = 1.7 \cdot \varepsilon_{sm} \cdot s_{rm} = w_k = 1.7 \cdot \varepsilon_{sm} \cdot s_{rm,th} \tag{1b}$$

If the design formulation for crack width given by new EC2 (2004) is used, the maximum crack spacing, $s_{r,max}$, can be substituted in Eqn.4 by the proposed expression (Eqn.13) of $s_{rm,th,k}$:

$$w_k = s_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm}) = w_k = s_{rm,th,k} \cdot (\varepsilon_{sm} - \varepsilon_{cm}) \tag{4}$$

In Figures 5-6 predictions of crack width given by Eqn.1b (Fig. 5a-b) and Eqn.4 (Fig. 6a-b) introducing the proposed formulations of crack spacing (Eqn.11 and Eqn.13) are compared with experimental crack widths belonging to Authors' database (Fig. 5a-6a) and to an extended database comprehensive of other experimental results (Yoshizawa and Wu 1999, Matthys 2000, Fig. 5b-6b).

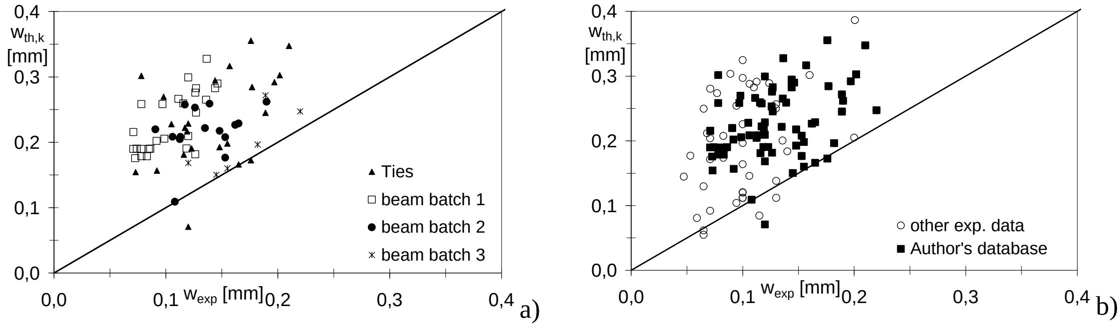

Fig. 5. Experimental vs. theoretical crack width by Eqns.1b, 2, 11: a) Authors' results b) Extended database

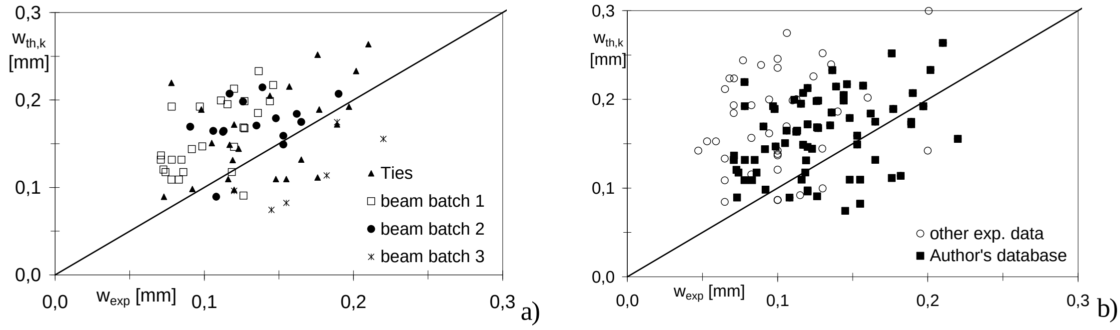

Fig. 6. Experimental vs. theoretical crack width by Eqns.4, 6, 13: a) Authors' results b) Extended database

In Table 2 the synthetic statistical parameters are extended also to design value of crack width calculated according to Eqn.1 and Eqn.4 using the proposed formulations for crack spacing and referring to the Authors' database. If the approach of past EC2 (1992) is adopted, crack width values proposed by Authors (Eqns.1b-11) show similar scatters respect to code provisions given by Eqns.1b-3 for RC elements (106% and 0.112mm vs. 117% and 0.125mm), while formulation of fib bulletin, specific for RC FRP strengthened elements, seems to be better (58% and 0.0067) due to the very lower prediction of crack spacing. On the contrary, if the approach of new EC2 (2004) is followed, that should be the mandatory one, the Authors' suggestion for design crack spacing (Eqn.13) introduced in Eqn.4 furnishes a sensible improvement for crack width provisions respect to formulation for RC elements (53% and 0.056mm vs. 98% and 0.112mm).

Table 1. Mean deviation of code provisions respect to the experimental results

	s_{rm} (EC2/92-Eqn.3)	$s_{r,max}$ (EC2/04-Eqn.5)	s_{rm} (fib-Eqn.7)	w_k (EC2/92-Eqn.1b)	w_k (fib-Eqn.1b)	w_{max} (EC2/04-Eqn.4)
n	43	43	32	70	53	70
$\sigma_{\%}$,	42%	175%	47%	117%	58%	98%
σ [mm]	25.0	103.0	41.0	0.125	0.067	0.112
$\bar{\delta}$	0.881	0.483	2.086	0.556	1.333	0.70
σ_{δ}	0.272	0.255	1.588	0.212	1.235	0.333

Table 2. Mean deviation of proposed provisions respect to the experimental results

	$s_{rm,th,m}$ (Eqn.11)	$s_{rm,th,k}$ (Eqn.13)	$w_{th,k}$ (Eqns.1b-11)	$w_{th,k}$ (Eqns.4-13)
n	43	43	70	70
$\sigma_{\%}$,	35%	92%	106%	53%
σ [mm]	22.0	53.6	0.112	0.056
$\bar{\delta}$	0.940	0.611	0.60	0.82
σ_{δ}	0.273	0.177	0.224	0.364

5 CONCLUSIONS

The experimental results of 32 RC beams and 12 RC ties strengthened with FRP laminates tested by the Authors have been collected in terms of crack spacing and widths. A lack of information is revealed about cracking performance, even if control of crack width remains a topic item for the durability of RC elements also when an FRP strengthening is applied, because the internal steel rebars still collaborate to the bearing capacity needing to be protected by corrosion phenomena.

The formulation to evaluate crack spacing proposed herein takes into account all the parameters influencing cracking phenomena and has been calibrated basing on an experimental database. The results are analyzed in order to estimate a characteristic value for crack spacing and the obtained formula appears to be more efficient than the codes ones, both in terms of mean and design values. Furthermore when the proposed expression of crack spacing is used for evaluating crack width according to the EC2 approach, the comparisons show about the same reliability of the original formulations of EC2 (1992) or fib bulletin (2001) for RC elements extended to strengthened ones. On the contrary if the new EC2 (2004) approach to calculate design values of crack width is used, Authors' provision seem give a better agreement. However it has to be underlined that following the EC2 approach, the problem is still governed by steel-concrete bond relationship.

Moreover a reliable provision of crack spacing can be useful not only for serviceability verifications, but also at ultimate conditions for studying intermediate crack debonding.

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