Performance prediction of eccentrically loaded RC columns wrapped with FRP

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ABSTRACT: This paper presents an analytical model for performance prediction of eccentrically loaded reinforced concrete (RC) columns wrapped with externally bonded fiber reinforced polymers (FRP). The proposed model is based on realistic materials' laws, and accounts for the change in geometry caused by the lateral deformation under eccentric loading. It also takes into consideration the confinement effect caused by FRP wrapping system at various load eccentricities. An experimental program was carried out to examine the model's accuracy. Test parameters included the confinement condition (No wrapping, full FRP-wrapping, and partial FRP-wrapping), and the eccentricity-to-section height (*e/h*) ratio (0.3, 0.43, 0.57, and 0.86). Research findings indicated that the strength gain caused by FRP wrapping system was inversely proportional to the eccentricity ratio. A comparison between the analytical and the experimental results has demonstrated the model's accuracy and validity.

1 INTRODUCTION

A large number of existing RC infrastructures in developed countries are suffering from distress due to overuse and/or inadequate maintenance. Structural strengthening is an economical solution, and hence frequently required to extend the functional service lives of deficient RC structures. Although reinforced concrete and grout-injected steel jacketing systems are effective in increasing the structural capacity, they are labor consuming and sometimes difficult to implement on site. Hence, an innovative, durable, easy-to-install, and cost effective strengthening system, such as externally bonded fiber reinforced polymer (FRP), is essentially required to replace outdated strengthening techniques. Numerous analytical models have been developed to predict the strength of concentrically loaded RC columns wrapped with FRP (Teng et al. 2002). In practical situations, RC columns are often, however, exposed to combined flexural and axial loading. The strain gradient caused by an eccentric loading results in a non-uniform confining pressure which would reduce the efficiency of the FRP wrapping system (Chaallal & Shahaway 2000; Parvin & Wang 2001). For eccentrically loaded RC columns, the secondary moment caused by the *P-Δ* effect would result in reducing the load carrying capacity, and hence account should be taken of the change in geometry caused by the lateral deformation. The present research is then initiated to develop a simple, yet accurate, analytical model that can predict the strength of eccentrically loaded RC columns wrapped with FRP. An experimental study was undertaken to examine the model's accuracy and validity.

2 MODEL DEVELOPMENT

2.1 *Materials laws*

Material constitutive models used in the present study are shown in Figure 1. The stress-strain relationship of an unconfined concrete in compression is described by a parabolic relationship.

$$f_c = f_c^{'} \left[\frac{2\varepsilon_c}{\varepsilon_{co}} - (\frac{\varepsilon_c}{\varepsilon_{co}})^2 \right] \tag{1}$$

Where f_c = concrete compression stress, $f_c^{'}$ = concrete compression strength, ε_c = concrete strain, and ε_{co} = concrete strain corresponding to $f_c^{'}$. The stress-strain model of an FRP-confined concrete is divided into two portions. The proposed model assumes that the ascending branch of the curve, between zero stress and a stress equal to $f_c^{'}$, is parabolic similar to that of the unconfined concrete (Samaan et al. 1998; Fam et al. 2003). Beyond $f_c^{'}$, the concrete stress is assumed to increase linearly until it reaches the confined strength of the concrete (Mirmiran et al. 1999; Teng et al. 2002). Based on extensive research (Teng et al. 2002), the strength and corresponding ultimate compressive strain of an FRP-confined concrete under concentric loading, $f_{cco}^{'}$ and ε_{ccu}, respectively, are given by:

$$f_{cco}^{'} = f_c^{'}(1 + 2\frac{f_l}{f_c^{'}}) \tag{2}$$

$$\varepsilon_{ccu} = \varepsilon_{co}(1.75 + 10\frac{f_l}{f_c^{'}}) \tag{3}$$
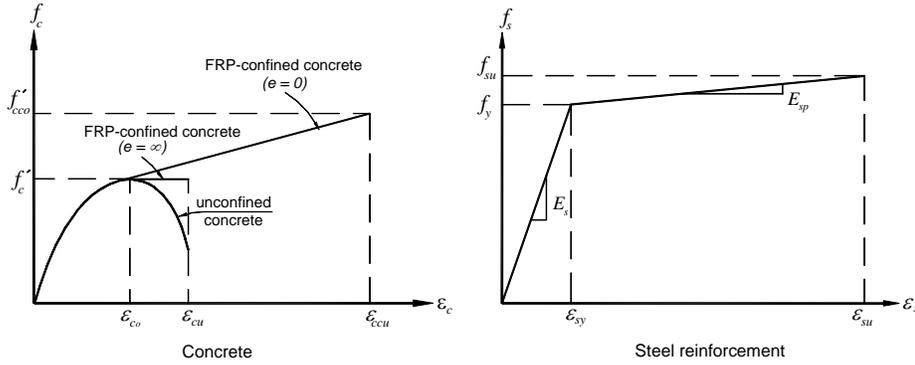
Figure 1. Materials models

Experimental test results showed that the gain in the ultimate compressive strain caused by FRP was inversely proportional to the eccentricity ratio (El Maaddawy 2008). Some researchers also reported an inverse relationship between the confined strength and the eccentricity ratio (Fam et al. 2003). Accordingly the strength and ultimate strain of the concrete under eccentric loading, $f_{cc}^{'}$ and ε_{cc}, respectively, are conservatively assumed to be dependent on e/h as follows:

$$f_{cc}^{'} = f_c^{'} + (f_{cco}^{'} - f_c^{'})(\frac{1}{1+e/h}) \tag{4}$$

$$\varepsilon_{cc} = \varepsilon_{cu} + (\varepsilon_{ccu} - \varepsilon_{cu})(\frac{1}{1+e/h}) \tag{5}$$

$$f_l = k_s \frac{2f_{fr}t_{fe}}{\frac{1}{2}(b+h)} \tag{6}$$

$$k_s = [1 - \frac{(b-2R_c)^2 + (h-2R_c)^2}{3A_g} - \rho_s]/(1-\rho_s) \tag{7}$$

Where f_l = the effective lateral confining pressure provided by FRP, ε_{cu} = ultimate compressive stain of an unconfined concrete, k_s = shape factor, f_{fr} = rupture strength of FRP, t_{fe} = effective thickness of FRP ($t_{fe} = t_f$ in case of full wrapping and $t_{fe} = t_f w_f/S_f$ in case of partial wrap-

ping), t_f = thickness of FRP, w_f = width of an FRP strip, S_f = center-to-center spacing between FRP strips, R_c = radius of the column's corner, b = width of section, h = height of section, e = load eccentricity, A_g = gross area of the section, and ρ_s = steel ratio. The stress-strain relationship of steel is idealized to be linear elastic-plastic with a post-yield strain hardening of 1 %.

2.2 *Compatibility and equilibrium requirements*

The strain and stress distributions for unconfined and FRP-confined concrete are shown in Figures 2a and b, respectively. The sectional curvature, ϕ, is then given by:

$$\phi = \frac{\varepsilon_{c,\max}}{c} \tag{8}$$

Where $\varepsilon_{c,max}$ = strain at the extreme compression fiber, and c = depth of the neutral axis. At failure, $\varepsilon_{c,max} = \varepsilon_{cu}$ for an unconfined concrete while for a confined concrete $\varepsilon_{c,max} = \varepsilon_{cc}$. Equilibrium condition is imposed in terms of axial force, P_n, and bending moment, M_n:

$$\alpha_1\beta_1 f_c' c_1 b + \frac{f_{cc}' + f_c'}{2} c_2 b + A_s' f_s' - A_s f_{st} = P_n \tag{9}$$

$$(\alpha_1\beta_1 f_c' c_1 b + \frac{f_{cc}' + f_c'}{2} c_2 b)(\frac{h}{2} - d_c) + A_s' f_s'(\frac{h}{2} - d') + A_s f_{st}(d - \frac{h}{2}) = M_n \tag{10}$$

$$d_c = c - \frac{(\alpha_1\beta_1 f_c' c_1)(c_1 - \frac{\beta_1 c_1}{2}) + (\frac{f_{cc}' + f_c'}{2} c_2)(c_1 + \frac{2f_{cc}' + f_c'}{3(f_{cc}' + f_c')} c_2)}{\alpha_1\beta_1 f_c' c_1 + \frac{f_{cc}' + f_c'}{2} c_2} \tag{11}$$

Where α_1 and β_1 = stress block factors (Collins and Mitchell 1987), c_1 = height of the parabolic portion of the concrete stress distribution, c_2 = height of the trapezoidal portion of the concrete stress distribution, d_c = distance between the extreme compression fiber and the resultant compression force in concrete, A_s' = area of compression steel, f_s' = stress in compression steel, A_s = area of tension steel, and f_s = stress in tension steel.
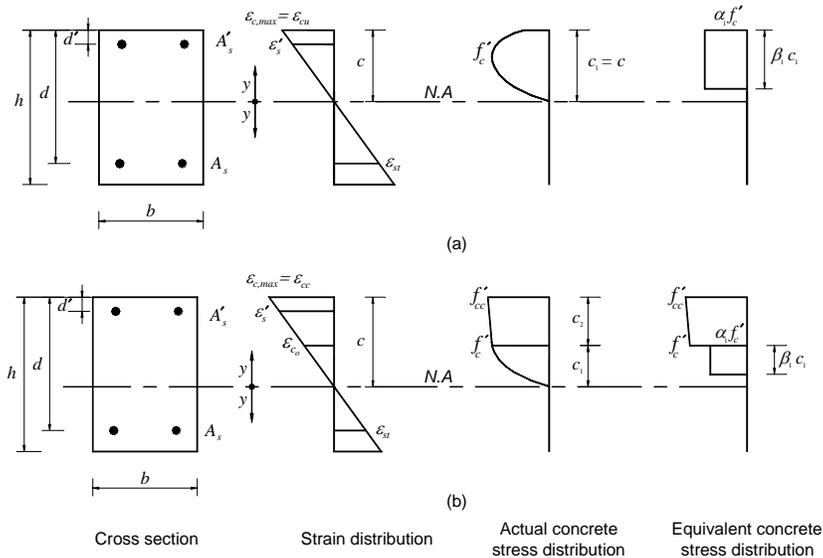
Figure 2. Strain and stress distributions

2.3 *Lateral mid-height displacement*

The moments, curvatures and deformations of an RC column under eccentric loading are shown in Figure 3. The lateral mid-height deflection, Δ, can be related to the mid-height curvature, ϕ, and the total length of the column, L, as follows:

$$\Delta = \frac{\phi L^2}{k_1} \tag{12}$$

The moment of inertia of each end corbel in the present study was 16 times the inertia of the column section in the test region, and hence it can be assumed that $k_1 = 12$.

2.4 *Modeling procedure*

The modeling procedure can be summarized as follows:

- For a given *e/h*, use the undeformed geometry to determine the sectional forces and curvature at mid-height of the column.
- Use the mid-height sectional curvature to calculate the lateral mid-height deflection.
- Modify the initial *e/h* to account for the lateral mid-height deflection.
- Use the modified *e/h* to recalculate the sectional forces and curvature at the mid-height.
- Return to step 2 if there is a significant change in the sectional forces.
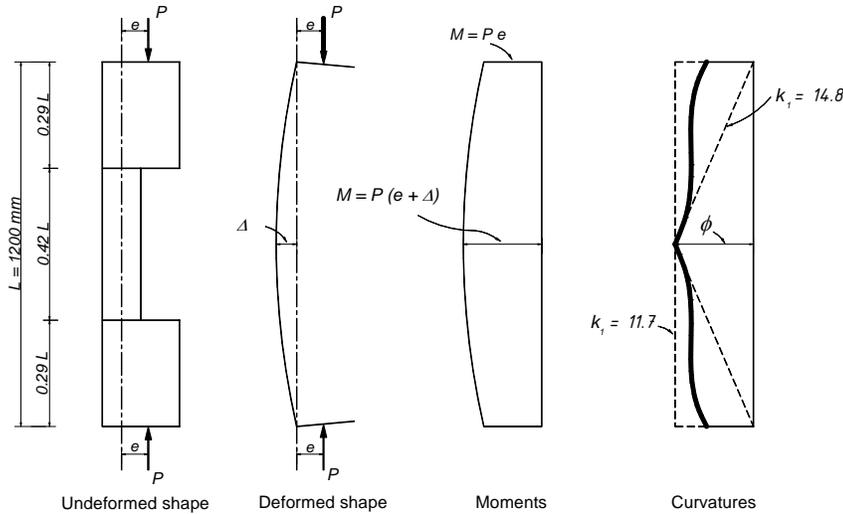
Figure 3. Deformed shape and corresponding moments and curvatures

3 MODEL VERIFICATION

An experimental program was carried out to examine the model's accuracy. Details of test specimen are given in Figure 4. A total of twelve specimens were tested. Test parameters included the confinement condition (No wrapping, full FRP-wrapping, and partial FRP-wrapping with $w_f/S_f = 0.6$), and the eccentricity ratio (0.3, 0.43, 0.57, and 0.86). The concrete strength was 28.5 MPa whereas the longitudinal steel had yield and ultimate strengths of 550 and 725 MPa, respectively. A cured carbon fiber reinforced polymer (CFRP) composite laminate used in the present study had a thickness of 0.381 mm, a tensile strength of 894 MPa, a tensile modulus of 65.4 GPa, and an ultimate elongation of 1.33 % (information was extracted from the manufacturer's data sheet). The columns corners were rounded to a radius of about 10 mm.
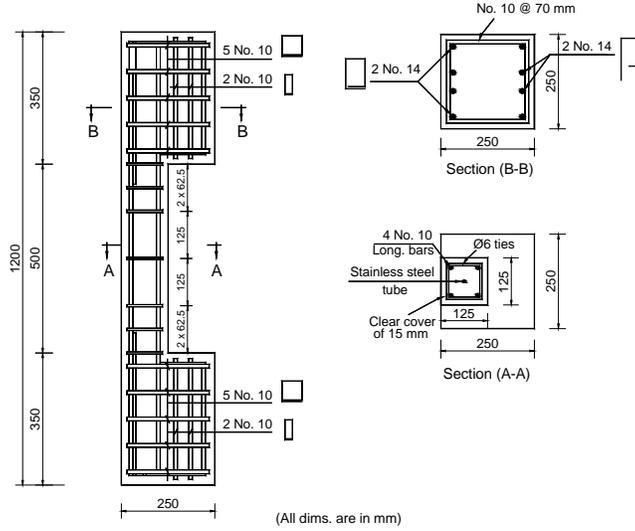
Figure 4. Specimen details (all dimensions are in mm)
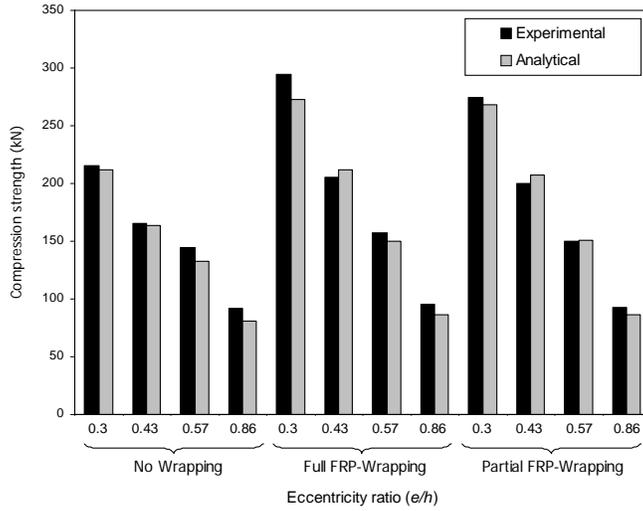
Figure 5. Analytical and experimental results

A comparison between the analytical and the experimental results are given in Figure 5. It can be seen that the strength gain caused by CFRP wrapping decreased as the eccentricity ratio was increased. One layer of full CFRP-wrapping resulted in about 37 % strength gain at a nominal *e/h* of 0.3 whereas only 3 % strength gain was recorded at a nominal *e/h* of 0.86. All predicted results were within 12 % error band which confirms the ability of the proposed model to accurately predict the strength of unwrapped and FRP-wrapped RC columns under eccentric loading. Figure 6 shows typical interaction curves for a square column having same cross sectional dimensions, steel reinforcement and materials properties as those of the specimen used in the present study. Three different wrapping schemes were considered. The column was either fully wrapped with one or two layers of CFRP, or partially wrapped with one layer of CFRP strips having a clear spacing $s_{clear} = w_f$. In these curves the column's strength under concentric loading, P_{no}, is predicted by:

$$P_{no} = \begin{cases} 0.85 f_c^{'}(A_g - A_{st}) + A_{st} f_{so} & \text{for } \text{unwrapped-columns} \\ f_{cco}^{'}(A_g - A_{st}) + A_{st} f_{sc} & \text{for } \text{CFRP-wrapped columns} \end{cases} \tag{13}$$

Where A_{st} = total area of steel, f_{so} = steel stress corresponding to a steel strain of ε_{co}, and f_{sc} = steel stress corresponding to a steel strain of ε_{ccu} but not greater than the steel ultimate stress. It is evident that the strength gain increases with the amount of CFRP. The CFRP wrapping has a more pronounced effect on the strength gain when a compression mode failure is dominated.
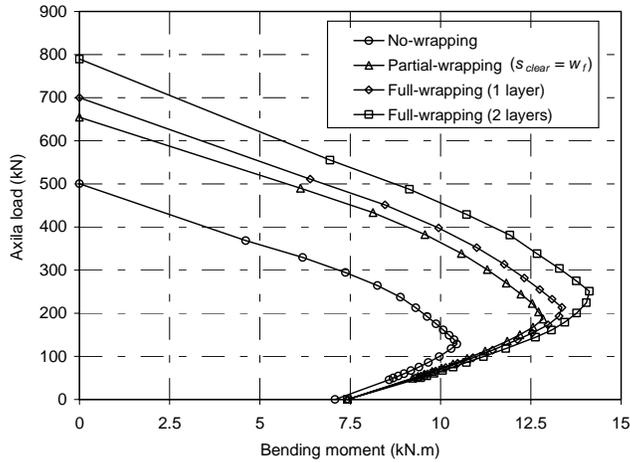
Figure 6. Typical interaction curves

4 CONCLUSIONS

An analytical model that can predict the strength of eccentrically loaded RC columns wrapped with FRP was developed and verified against test results. The model accounts for the confinement effect of FRP wrapping and the change in geometry caused by the lateral deformation under eccentric loading. Research findings indicate that the strength gain caused by FRP decreases as the eccentricity ratio is increased.

ACKNOWLEDGMENTS

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